

Agenda/Objectives/Notes PAPPC Section 2.6

Starter Problem

Simplify the difference quotient: $\frac{\sqrt{x+h+1} - \sqrt{x+1}}{h}$

Today's Agenda

1. Starter Problem
2. Review assignment due
3. Today's objectives
4. Today's assignment: Read Section 2.6, 193/2, 6, 11, 16, 18, 19, 24, 25, 33, 39, 46, 48, 53, 60, 63, 74, 77, 81, 82, 87, 88, 91

Today's Objectives: You will be able to

1. Find the domains of rational functions.
2. Find the horizontal and vertical asymptotes of rational functions.
3. Analyze and sketch the graphs of rational functions.
4. Use rational functions to model and solve real-life problems

Notes/Examples

Definition of Vertical and Horizontal Asymptotes

1. The line $x = a$ is a **vertical asymptote** of the graph of f if $f(x) \rightarrow \infty$ or $f(x) \rightarrow -\infty$ as $x \rightarrow a$, either from the left or from the right
2. The line $y = b$ is a **horizontal asymptote** of the graph of f if $f(x) \rightarrow b$ as $x \rightarrow \infty$ or $x \rightarrow -\infty$

Asymptotes of a Rational Function

$$\text{Let } f(x) = \frac{N(x)}{D(x)}$$

1. The graph of f has a *vertical* asymptote for at the zeros of $D(x)$.
2. The graph of f has one or no *horizontal* asymptote determined by comparing the degrees of $N(x)$ and $D(x)$.
 - a. if the degree of $N(x)$ is $>$ the degree of $D(x)$, then there is no horizontal asymptote.
 - b. if the degree of $N(x)$ is $<$ the degree of $D(x)$, then the equation for the horizontal asymptote is $y = 0$.

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- c. if the degree of $N(x)$ = the degree of $D(x)$, then the equation for the horizontal asymptote is $y = \frac{\text{coefficient of lead term of } N(x)}{\text{coefficient of lead term of } D(x)}$.

To graph a rational function:

1. Simplify the function if possible
2. Find and plot the y -intercept (if any) by evaluating the $f(0)$.
3. Find the zeros of the numerator (if any) by solving the equation $N(x) = 0$. Then plot the corresponding x -intercepts.
4. Find the zeros of the denominator (if any) by solving the equation $D(x) = 0$. Then sketch the corresponding vertical asymptotes.
5. Find and sketch the horizontal asymptote (if any) by using the rule for finding horizontal asymptotes.
6. Find at *least* one point between the x -intercepts and one point beyond the vertical asymptotes.
7. Use smooth curves to complete the graph between and beyond the vertical asymptote(s).

Be prepared to discuss the method used to find *slant* asymptotes.